Comparative Analysis of Advanced Time Series Forecasting Techniques: Evaluating the Accuracy of ARIMA, Prophet, and Deep Learning Models for Predicting Inflation Rates, Exchange Rates, and Key Financial Indicators

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Abstract

This paper presents a rigorous comparative analysis of advanced time series forecasting techniques, specifically focusing on the application of ARIMA, Prophet, and deep learning models to the prediction of key financial indicators, including inflation rates and exchange rates. The study seeks to address the growing demand for accurate and reliable forecasting methods in financial markets, where precise predictions are crucial for informed decisionmaking. By systematically evaluating the performance of these models, this research contributes to the ongoing discourse on the efficacy of traditional versus modern forecasting approaches in the context of financial time series data.

The analysis begins with an exploration of the theoretical foundations and mathematical formulations underlying each model. ARIMA, a widely used statistical method, is known for its capacity to model linear relationships in time series data by capturing autoregressive, differencing, and moving average components. Prophet, developed by Facebook, is a relatively recent addition to the forecasting landscape, designed to handle seasonality, holidays, and trends with a focus on ease of use and interpretability. The deep learning models, on the other hand, represent a paradigm shift, employing neural networks to capture complex, nonlinear relationships in time series data. These models, including Long ShortTerm Memory (LSTM) and Convolutional Neural Networks (CNNs), have demonstrated significant potential in various domains but require extensive computational resources and expertise.

In this study, we implement these models on a comprehensive dataset comprising historical inflation rates, exchange rates, and other critical financial metrics. The dataset is pre-processed to ensure consistency and to account for missing values, outliers, and non-stationarities, which are common challenges in financial time series analysis. Each model is trained and tested using a rolling window approach, allowing for the evaluation of forecast accuracy over multiple time horizons. The models are assessed based on several performance metrics, including Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE), providing a multifaceted view of their predictive capabilities.

The findings reveal distinct strengths and weaknesses across the models. ARIMA, while robust in capturing linear trends and seasonality, exhibits limitations in handling non-linear patterns and sudden shifts in the data, which are often encountered in financial markets. Prophet, with its flexible handling of seasonality and trend components, shows promise, particularly in cases where domain knowledge can be leveraged to improve model specification. However, its performance is sensitive to the choice of hyperparameters and may require extensive tuning for optimal results. The deep learning models, particularly LSTM, excel in capturing complex, non-linear relationships and demonstrate superior performance in scenarios with high volatility and noise. However, their computational demands and susceptibility to overfitting present challenges, particularly in situations with limited data or when interpretability is a priority.

The study also delves into the computational efficiency of these models, an important consideration for real-time forecasting applications. ARIMA, being relatively simple in its formulation, is computationally efficient and suitable for scenarios with limited computational resources. Prophet, while more computationally intensive than ARIMA, offers a reasonable trade-off between accuracy and efficiency, making it a viable option for many practical applications. The deep learning models, despite their superior accuracy in complex scenarios, require substantial computational resources and longer training times, which may limit their applicability in time-sensitive or resource-constrained environments.

Furthermore, the paper explores the models' ability to handle seasonality, trends, and noise, which are critical features of financial time series. ARIMA's reliance on differencing to achieve stationarity may lead to the loss of important information, particularly in series with complex seasonal patterns. Prophet's explicit modeling of seasonality and holidays provides an advantage in such cases, allowing for more accurate forecasts in the presence of recurrent patterns. The deep learning models, with their capacity to learn from large amounts of data, are particularly adept at handling noise and uncovering hidden patterns, making them wellsuited for highly volatile financial time series.

Keywords

time series forecasting, ARIMA, Prophet, deep learning models, financial indicators, inflation rates, exchange rates, computational efficiency, seasonality, non-linear relationships.

1. Introduction

1.1 Background and Motivation

In the realm of economic decision-making, the ability to accurately forecast key financial indicators such as inflation rates, exchange rates, and other critical financial metrics is of paramount importance. These forecasts serve as the foundation upon which a wide range of economic policies, investment strategies, and business decisions are formulated. In an increasingly complex and volatile global economy, the precision of financial forecasting has become even more critical, as it directly influences the stability and growth prospects of economies, markets, and institutions. The inherent uncertainty and dynamic nature of financial markets, characterized by sudden shifts and non-linear patterns, present significant challenges to forecasting models, necessitating the use of sophisticated techniques capable of capturing these complexities.

Traditional time series models, such as the Autoregressive Integrated Moving Average (ARIMA), have long been the cornerstone of financial forecasting due to their robustness in modeling linear relationships and their well-established theoretical foundations. However, the limitations of these models in handling non-linearities, abrupt changes, and the intricate seasonality often observed in financial time series have spurred the development of more advanced techniques. The advent of Prophet, a model specifically designed to handle seasonal effects, holidays, and long-term trends with minimal user intervention, represents a significant advancement in the field. Its ability to incorporate domain-specific knowledge into the forecasting process makes it particularly appealing for practical applications in finance.

Concurrently, the rise of deep learning models has introduced a new paradigm in time series forecasting. These models, including Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), have demonstrated remarkable success in capturing complex, non-linear relationships in data across various domains. Their application to financial time series, where such non-linearities are prevalent, has shown great promise. However, the complexity and resource-intensive nature of these models, coupled with challenges related to overfitting and interpretability, necessitate a thorough examination of their suitability for financial forecasting.

The motivation for this research is rooted in the need to provide a comprehensive comparative analysis of these diverse forecasting techniques, each representing a different approach to modeling financial time series. By systematically evaluating their performance in predicting key financial indicators, this study aims to offer valuable insights into the relative strengths and limitations of each model. The findings of this research are intended to guide practitioners in selecting the most appropriate forecasting approach for their specific needs, thereby enhancing the accuracy and reliability of financial forecasts in an increasingly uncertain economic environment.

1.2 Research Objectives

The primary objective of this study is to conduct a detailed comparative analysis of three distinct time series forecasting techniques: ARIMA, Prophet, and deep learning models. This research seeks to evaluate the efficacy of these models in forecasting critical financial indicators, including inflation rates and exchange rates, which are integral to economic planning and decision-making. The focus will be on assessing the accuracy, computational efficiency, and ability of each model to handle the complexities inherent in financial time series data, such as seasonality, trends, and noise.

The first objective is to implement and fine-tune the ARIMA model, a traditional statistical approach, to serve as a benchmark for comparison. Despite its widespread use, ARIMA's limitations in capturing non-linearities and handling complex seasonal patterns necessitate a rigorous evaluation in the context of modern financial data. The second objective involves the application of the Prophet model, which is designed to automatically handle seasonal effects and incorporate domain-specific events, providing an innovative solution to some of the limitations inherent in traditional models. This research will explore the extent to which Prophet's flexibility and ease of use translate into improved forecasting accuracy for financial indicators.

The third objective centers on the deployment of deep learning models, with a particular focus on LSTM and CNN architectures. These models, which leverage neural networks to capture complex, non-linear relationships, represent the cutting edge of time series forecasting. The study will assess their performance in forecasting financial time series, with particular attention to their ability to model volatility, noise, and sudden shifts in data patterns. Additionally, the research will explore the computational demands of these models, evaluating their practicality for real-time forecasting applications in finance.

An integral part of the research is the systematic evaluation of each model's performance using a comprehensive set of financial data. This evaluation will be conducted using a rolling window approach, allowing for a dynamic assessment of forecast accuracy over multiple time horizons. The study will employ a range of performance metrics, including Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE), to provide a nuanced understanding of each model's strengths and weaknesses.

The ultimate goal of this research is to provide actionable insights that can inform the selection of forecasting techniques in financial practice. By identifying the conditions under which each model excels or falters, this study aims to contribute to the optimization of financial forecasting processes, ultimately enhancing the quality and reliability of economic decisionmaking. The findings are expected to be particularly relevant for financial analysts, policymakers, and researchers seeking to leverage advanced time series forecasting techniques to navigate the complexities of modern financial markets.

2. Literature Review

2.1 Traditional Time Series Models

Traditional time series models have long been the cornerstone of financial forecasting, with the Autoregressive Integrated Moving Average (ARIMA) model being one of the most widely utilized techniques. ARIMA, first introduced by Box and Jenkins in the 1970s, is a linear model that captures the autocorrelations within a time series by combining autoregression (AR), differencing to achieve stationarity (I), and a moving average (MA) of past errors. The model is particularly valued for its theoretical rigor and ability to provide reliable forecasts in cases where the underlying data exhibits a linear relationship. The ARIMA model operates under the assumption of stationarity, meaning that the statistical properties of the series, such as mean and variance, remain constant over time. This assumption necessitates preprocessing steps such as differencing or transformation to stabilize the variance and ensure stationarity.

Despite its robustness in handling univariate time series data, ARIMA has several limitations when applied to financial data. Financial time series often exhibit non-linear patterns, structural breaks, and volatility clustering—features that ARIMA models struggle to capture effectively. Additionally, the model's reliance on historical data implies that it may not perform well in the presence of sudden market shifts or external shocks, which are common in financial markets. Extensions of ARIMA, such as Seasonal ARIMA (SARIMA) and ARIMA with exogenous variables (ARIMAX), have been developed to address seasonality and incorporate external factors, respectively. However, these extensions add to the complexity of model identification and parameter estimation, making them less practical for real-time forecasting.

Other traditional models, such as Exponential Smoothing and Holt-Winters methods, have also been extensively used in financial forecasting. Exponential Smoothing models apply weighted averages to past observations, where the weights decay exponentially over time. The Holt-Winters method, an extension of Exponential Smoothing, includes components for trend and seasonality, making it suitable for data with regular patterns. However, like ARIMA, these models are limited in their ability to handle non-linearity and are primarily useful for short-term forecasts in stable environments.

The limitations of traditional time series models, particularly in capturing the complexities of financial data, have spurred the development of more advanced forecasting techniques. As financial markets become more volatile and interconnected, the need for models that can account for non-linearities, structural breaks, and high-frequency data has become increasingly apparent. This has led to the emergence of modern forecasting approaches, which seek to address the shortcomings of traditional models.

2.2 Modern Forecasting Approaches

The modern era of time series forecasting has witnessed the advent of models that incorporate more sophisticated techniques to handle the complexities inherent in financial data. One such model is Prophet, developed by Facebook in 2017. Prophet is a decomposable time series model that was specifically designed to handle seasonality, holidays, and long-term trends with minimal user intervention. Unlike ARIMA, which requires extensive data preprocessing and manual parameter tuning, Prophet automates much of the forecasting process. The model is based on an additive regression model where the observed time series is decomposed into trend, seasonality, and holiday effects. Prophet's ability to incorporate domain-specific knowledge, such as known holidays or special events, makes it particularly suited for business and financial forecasting where such factors can significantly impact the data.

Prophet's development was motivated by the need for a model that is both robust to missing data and capable of handling outliers and abrupt changes in trends. These features make it highly applicable to financial time series, which are often subject to sudden shifts due to market events or policy changes. The model's flexibility and ease of use have led to its widespread adoption in various industries, including finance, where it is used for tasks such as demand forecasting, revenue prediction, and risk management.

Parallel to the development of models like Prophet, the rise of deep learning has introduced a new paradigm in time series forecasting. Deep learning models, particularly those based on Recurrent Neural Networks (RNNs) and their variants like Long Short-Term Memory (LSTM) networks, have shown significant promise in capturing complex, non-linear relationships in time series data. LSTMs, in particular, are designed to overcome the vanishing gradient problem commonly associated with traditional RNNs, making them well-suited for capturing long-term dependencies in data. This is particularly relevant for financial time series, where the impact of past events can persist over extended periods.

Convolutional Neural Networks (CNNs), though originally designed for image processing tasks, have also been adapted for time series forecasting. CNNs are capable of capturing local patterns in data through their hierarchical structure, making them effective in identifying and modeling short-term fluctuations in financial time series. When combined with LSTMs in hybrid models, CNNs can enhance the model's ability to capture both short-term and longterm patterns, leading to more accurate forecasts.

The application of deep learning models to financial forecasting has been driven by the increasing availability of high-frequency data and advancements in computational power. These models, however, are not without their challenges. Deep learning models are often criticized for their black-box nature, making it difficult to interpret the results and understand the underlying mechanisms driving the forecasts. Additionally, the computational intensity and the need for large datasets pose practical challenges, particularly in real-time forecasting scenarios.

Despite these challenges, the potential of deep learning models in financial forecasting is undeniable. Their ability to model complex, non-linear relationships and adapt to changing patterns in data makes them a powerful tool in the arsenal of modern financial forecasting techniques. The next section will review existing comparative studies that have sought to evaluate the performance of these traditional and modern forecasting approaches, identifying gaps that this research aims to address.

2.3 Comparative Studies in Financial Forecasting

The comparative evaluation of forecasting techniques has been a topic of significant interest in the literature, particularly in the context of financial time series. Numerous studies have sought to compare the performance of traditional models like ARIMA with modern approaches such as machine learning and deep learning models. These studies typically focus on key performance metrics such as accuracy, computational efficiency, and the ability to handle various characteristics of financial time series, including seasonality, trends, and noise.

Early comparative studies often highlighted the strengths of traditional models like ARIMA in terms of simplicity and interpretability. These models were found to perform well in stable environments where the underlying data exhibited linear patterns. However, as financial markets became more volatile and complex, the limitations of these models became increasingly apparent. Subsequent studies began to explore the potential of machine learning techniques, including Support Vector Machines (SVMs) and decision trees, which were shown to outperform traditional models in certain contexts, particularly in handling non-linearities.

The introduction of deep learning models into the comparative landscape marked a significant shift in the literature. Studies have demonstrated that deep learning models, particularly LSTMs, can achieve superior accuracy in forecasting tasks involving complex, non-linear time series. These models have been particularly effective in capturing long-term dependencies and handling the volatility that characterizes financial markets. However, the results have not been uniformly positive, with some studies highlighting issues related to overfitting, high computational costs, and the black-box nature of deep learning models.

Prophet, being a relatively recent addition to the forecasting toolbox, has also been the subject of comparative studies, particularly in business and financial contexts. These studies have generally found that Prophet performs well in scenarios where the data exhibit strong seasonal patterns or where domain-specific knowledge can be incorporated into the forecasting process. However, there is a relative paucity of studies that directly compare Prophet with deep learning models, particularly in the context of financial time series. This gap in the literature represents a significant opportunity for further research.

The existing literature also highlights a need for more comprehensive evaluations that consider a broader range of performance metrics. While accuracy is often the primary focus, factors such as computational efficiency, scalability, and ease of use are also critical, particularly in real-world applications. Moreover, there is a need for studies that evaluate the robustness of different models across various financial contexts, including different asset classes, time horizons, and market conditions.

This research aims to address these gaps by providing a detailed comparative analysis of ARIMA, Prophet, and deep learning models in the context of financial forecasting. By evaluating these models across a comprehensive set of performance metrics and applying them to a diverse set of financial indicators, this study will provide valuable insights into the relative strengths and limitations of each approach. The findings are expected to contribute to the ongoing debate in the literature and provide practical guidance for financial practitioners seeking to enhance the accuracy and reliability of their forecasts.

3. Theoretical Foundations

3.1 ARIMA Model

The Autoregressive Integrated Moving Average (ARIMA) model stands as one of the most extensively studied and applied models in time series forecasting. Developed as a synthesis of autoregressive (AR) models and moving average (MA) models, ARIMA introduces an additional layer of complexity by incorporating differencing operations to render nonstationary data stationary. The stationarity assumption is crucial in time series analysis, as it implies that the statistical properties of the series, such as the mean and variance, remain constant over time. The ARIMA model, therefore, provides a robust framework for forecasting in scenarios where the data exhibit linear patterns and where stationarity can be reasonably achieved through differencing.

The mathematical formulation of the ARIMA model can be expressed as ARIMA(p, d, q), where the parameters p, d, and q represent the orders of the autoregressive, differencing, and moving average components, respectively. The general form of the ARIMA model can be represented as follows:

Φp(B)(1−B)dyt=Θq(B)ϵt

where yty_tyt denotes the observed time series at time ttt, BBB is the backshift operator such that Bkyt=yt−k, Φp(B)\Phi_p(B)Φp(B) and Θq(B)\Theta_q(B)Θq(B) are polynomials of orders p and q in the backshift operator B, and $\epsilon t \epsilon$ represents a white noise error term.

The autoregressive (AR) component is captured by the polynomial $\Phi p(B)$, which takes the form:

$$
\Phi p(B)=1-\varphi 1B-\varphi 2B2-\cdots -\varphi pBp
$$

where ϕ1,ϕ2,…,ϕp are the autoregressive coefficients. The AR component models the relationship between an observation and a specified number of lagged observations. This means that the current value of the time series is expressed as a linear function of its past values, with the strength of the relationship determined by the coefficients ϕi\phi_iϕi.

The differencing component, represented by $(1-B)^d$, is responsible for transforming the original non-stationary series into a stationary one. The differencing operation essentially subtracts the previous observation from the current observation, thereby removing trends or seasonality that could distort the forecasting model. The parameter d indicates the number of times the differencing operation is applied. For instance, if d=1, the model uses first-order differencing; if d=2, second-order differencing is applied, and so on. The choice of d is critical, as over-differencing can lead to a loss of valuable information, while under-differencing may leave residual non-stationarity in the series.

The moving average (MA) component, encapsulated by the polynomial $\Theta(q|B)$, is defined as:

$$
\Theta q(B)=1+\theta 1B+\theta 2B2+\cdots+\theta qBq
$$

where $\theta_1, \theta_2, \theta_2, \dots, \theta_k$ \theta_2, \dots, \theta_q $\theta_1, \theta_2, \dots, \theta_k$ are the moving average coefficients. The MA component models the relationship between an observation and a residual error from a moving average model applied to lagged observations. In essence, the

MA component accounts for the noise in the time series, smoothing out random fluctuations and enhancing the model's predictive accuracy.

The estimation of the ARIMA model parameters involves several steps, beginning with the identification of the appropriate orders of the AR, MA, and differencing components. This is typically done through an analysis of the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots, which provide insights into the dependencies in the time series. The estimation of the model parameters is often performed using maximum likelihood estimation (MLE), a method that seeks to find the parameter values that maximize the likelihood function given the observed data.

One of the key assumptions underlying the ARIMA model is the linearity of the relationship between the lagged values and the current observation. This linearity assumption, while simplifying the modeling process, also limits the ARIMA model's applicability in scenarios where the time series exhibits non-linear patterns. Moreover, the ARIMA model assumes homoscedasticity, meaning that the variance of the residuals remains constant over time. In financial time series, where volatility clustering and heteroscedasticity are common, this assumption may not hold, leading to potential inaccuracies in the forecasts.

Another critical aspect of the ARIMA model is its sensitivity to the choice of parameters. The process of identifying the optimal orders of ppp, ddd, and qqq often involves trial and error, guided by diagnostic tests such as the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC). These criteria penalize the likelihood of the model by the number of estimated parameters, helping to avoid overfitting. However, the reliance on these criteria introduces subjectivity into the model selection process, which can be a drawback in highly automated forecasting environments.

Despite these limitations, the ARIMA model remains a foundational tool in time series analysis, particularly in scenarios where the data exhibit stable, linear trends. Its ability to provide interpretable models and its extensive theoretical foundation make it a preferred choice for many forecasting applications. However, as the complexity and volatility of financial time series continue to increase, the limitations of the ARIMA model become more apparent, necessitating the exploration of more advanced forecasting techniques such as Prophet and deep learning models. The next sections of this paper will delve into these modern approaches, providing a comparative analysis of their theoretical foundations, performance, and applicability in financial forecasting.

3.2 Prophet Model

The Prophet model, developed by Facebook's Core Data Science team, represents a significant advancement in time series forecasting, particularly in its ability to handle irregularities and non-linear trends in data. Unlike traditional models such as ARIMA, which rely heavily on the assumption of stationarity and linearity, Prophet is designed to address the complexities inherent in real-world time series data, including seasonality, holidays, and trend changes. The model's flexibility and interpretability have made it an increasingly popular tool for forecasting in various domains, especially where data exhibits pronounced seasonality and trend components.

The underlying mechanics of the Prophet model are rooted in the decomposition of the time series into three primary components: trend, seasonality, and holidays or events. Each component is modeled separately and then combined to generate the final forecast. This approach allows Prophet to capture a wide range of temporal patterns, making it particularly effective in scenarios where traditional models may struggle.

The trend component in Prophet is modeled using a piecewise linear or logistic growth model. The linear growth model assumes that the time series grows or declines at a constant rate, with the possibility of changes in the growth rate at specified points in time, known as changepoints. The logistic growth model, on the other hand, assumes that the growth of the time series follows an S-shaped curve, which is particularly useful in scenarios where there is a natural upper or lower bound on the data, such as population growth or market saturation. The trend component can be mathematically represented as follows:

$$
g(t) = (\delta +_{j=1} m \sum ajI(t \ge sj)) \cdot t
$$

In this equation, g(t) represents the trend at time t, δ is the base growth rate, a_j represents the rate change at each changepoint sj, and $I(t\geq s)$ is an indicator function that is equal to 1 when ttt is greater than or equal to the changepoint s_i and 0 otherwise. The sum of the rate changes and the base growth rate defines the overall trend in the data. The model automatically detects potential changepoints by analyzing the historical data, making it highly adaptive to sudden shifts in the underlying trend.

The seasonality component in Prophet is modeled using Fourier series, which allows for the capture of complex seasonal patterns with different periodicities. Prophet supports multiple seasonalities, such as daily, weekly, and yearly patterns, making it particularly effective in applications where the data exhibits multiple recurring cycles. The seasonality component is expressed as:

s(t)= $\sum_{k=1}^N$ (a_kcos(2πkt/P)+b_ksin(2πkt/P))

In this expression, s(t) represents the seasonal effect at time t, P is the period of the seasonality (e.g., 365.25 days for yearly seasonality), and a_k and b_k are the Fourier coefficients. The model determines the number of Fourier terms N based on the complexity of the seasonality, allowing it to capture both simple and complex seasonal patterns. This flexibility is a significant advantage over traditional time series models, which often require manual specification of seasonality or struggle to capture multiple seasonal cycles.

The third component of the Prophet model is the holiday or event effect, which accounts for the impact of known holidays, special events, or other external factors that can cause abrupt changes in the time series. The model allows users to specify a list of holidays and their corresponding effects on the time series. Prophet then models these effects using an indicator variable, which is 1 when the holiday occurs and 0 otherwise. The holiday component is particularly useful in retail, finance, and other industries where sales, prices, or other metrics are influenced by recurring events.

The overall Prophet model can be represented as:

$$
y(t)=g(t)+s(t)+h(t)+\epsilon t
$$

where $y(t)$ is the observed value at time t, $g(t)$ represents the trend component, $s(t)$ represents the seasonality component, h(t) represents the holiday or event effect, and ϵ_t is the error term, assumed to be normally distributed with mean 0 and variance σ2.

One of the unique features of the Prophet model is its ability to incorporate prior knowledge through the use of Bayesian structural time series. This approach allows users to specify prior distributions for the model parameters, such as the growth rate or the seasonal amplitude, based on domain expertise or historical data. The model then combines this prior knowledge with the observed data to produce more accurate and interpretable forecasts. This capability is particularly valuable in financial forecasting, where expert knowledge of market behavior, economic cycles, or regulatory changes can significantly enhance the accuracy of the predictions.

Another distinguishing feature of Prophet is its emphasis on interpretability and userfriendliness. The model's parameters, such as the growth rate, seasonal effects, and holiday impacts, are easily interpretable, making it accessible to non-experts while still providing the flexibility needed by advanced users. Additionally, Prophet's implementation is designed to be robust to missing data, outliers, and irregular time intervals, which are common challenges in real-world time series forecasting. The model automatically handles these issues, allowing users to focus on the substantive aspects of their forecasting tasks.

3.3 Deep Learning Models

Deep learning has emerged as a transformative approach in the domain of time series forecasting, offering unparalleled capabilities in capturing complex patterns and dependencies that traditional statistical models may fail to recognize. The application of deep learning techniques to time series forecasting leverages the powerful feature extraction and learning capabilities of neural networks, which are particularly effective in modeling nonlinear relationships, temporal dependencies, and high-dimensional data. Among the various architectures employed in deep learning for time series analysis, Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs) have proven to be especially effective.

Long Short-Term Memory (LSTM) Networks

LSTM networks, a specialized form of Recurrent Neural Networks (RNNs), have gained significant attention in time series forecasting due to their ability to model long-term dependencies within sequential data. Unlike standard RNNs, which struggle with the vanishing gradient problem during training, LSTMs incorporate a memory cell and a set of gating mechanisms—input, output, and forget gates—that regulate the flow of information through the network. This architecture enables LSTMs to selectively retain or discard information over extended periods, making them particularly well-suited for time series data where the influence of past events may persist over long horizons.

The fundamental building block of an LSTM network is the memory cell, which maintains a state vector c_t at time step t. The memory cell is updated based on the input x_t , the previous hidden state h_{t−1}, and the current state c_{t-1} . The gating mechanisms control how information is incorporated into the memory cell and how it influences the output. The mathematical formulation of the LSTM cell is as follows:

> $f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$ $i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$ $c \sim_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$ $c_t = f_t \cdot c_{t-1} + i_t \cdot c_{t}$

 $o_t = σ(W_0 \cdot [h_{t-1}, x_t] + b_0)$

$h_t = o_t \cdot \tanh(c_t)$

Here, f_t represents the forget gate, i_t the input gate, o_t the output gate, and c_t the candidate cell state. W_f , W_i , W_c , and W_o are weight matrices, b_f , b_i , b_c , and b_o are bias vectors, and σ denotes the sigmoid activation function. The forget gate f_t determines the proportion of the previous cell state c_{t−1} that should be retained, the input gate i_t controls the extent to which new information c_t is added to the cell state, and the output gate o_t governs the output of the cell at time step t, which forms the hidden state h_t . The combination of these gating mechanisms allows LSTMs to maintain and utilize relevant information over long sequences, making them highly effective for forecasting tasks where both short-term fluctuations and long-term trends are crucial.

LSTM networks have been successfully applied to various financial forecasting tasks, including predicting stock prices, exchange rates, and economic indicators. Their ability to capture temporal dependencies and non-linear patterns makes them a powerful tool for modeling the complex dynamics of financial markets. However, the effectiveness of LSTM networks is contingent upon careful hyperparameter tuning and sufficient training data, as they are prone to overfitting when applied to small datasets or when the model complexity is not appropriately controlled.

Convolutional Neural Networks (CNNs)

While CNNs are traditionally associated with image processing, they have also been adapted for time series forecasting, particularly in scenarios where local patterns and feature extraction play a critical role. CNNs are designed to automatically and hierarchically learn features from raw input data, making them well-suited for tasks that involve pattern recognition and spatial hierarchies. In the context of time series forecasting, CNNs are employed to capture local dependencies within the time series, effectively identifying important patterns that may be indicative of future values.

The core operation of a CNN is the convolution, where a filter or kernel slides over the input data, performing element-wise multiplication and summation to produce a feature map. This operation allows CNNs to detect local patterns within the time series, such as abrupt changes, recurring cycles, or outliers. The mathematical representation of the convolution operation for a one-dimensional time series is as follows:

$y_i = \sum_{k=1}^{m} x_{i+k-1} \cdot w_k + b$

In this equation, yi represents the output at position i of the feature map, xi+k−¹ is the input time series, w_k is the filter, mmm is the filter size, and b is the bias term. The filter slides across the time series, capturing relevant local patterns that are then passed through non-linear activation functions, such as ReLU, to introduce non-linearity into the model.

CNNs also employ pooling layers, which downsample the feature maps to reduce dimensionality and computational complexity, while preserving the most critical information. The combination of convolutional and pooling layers enables CNNs to effectively capture both short-term and long-term dependencies in time series data, making them particularly useful in scenarios where local patterns or anomalies play a significant role in forecasting.

One of the notable advantages of CNNs in time series forecasting is their ability to handle high-dimensional data, such as multivariate time series, where multiple variables are observed over time. CNNs can process these variables simultaneously, learning the interdependencies and joint patterns that may be predictive of future outcomes. This capability is particularly valuable in financial forecasting, where multiple economic indicators, market indices, and other variables are often analyzed together to generate more accurate predictions.

CNNs have been applied to a wide range of financial forecasting tasks, including volatility prediction, trading signal generation, and risk assessment. Their ability to learn hierarchical representations of time series data makes them a powerful tool for capturing the intricate patterns and relationships that underlie financial markets. However, similar to LSTMs, CNNs require substantial computational resources and careful design to avoid overfitting and to ensure that the model generalizes well to unseen data.

Hybrid Models and Integration of LSTM and CNN Architectures

In recent years, there has been a growing interest in hybrid models that combine the strengths of LSTM and CNN architectures for time series forecasting. These hybrid models leverage the sequential modeling capabilities of LSTMs and the feature extraction capabilities of CNNs, resulting in models that can capture both local patterns and long-term dependencies in the data. The integration of LSTM and CNN layers in a single model allows for the simultaneous processing of temporal and spatial information, making these hybrid models particularly effective in complex forecasting tasks.

For example, a hybrid LSTM-CNN model may begin with a series of convolutional layers that extract local features from the time series, followed by LSTM layers that model the temporal dependencies among these features. This approach enables the model to capture fine-grained patterns that may be predictive of future values while maintaining the ability to model longterm trends and dependencies. Hybrid models have shown promising results in various financial forecasting applications, outperforming standalone LSTM or CNN models in terms of accuracy and robustness.

Deep learning models, particularly LSTM and CNN architectures, represent a significant advancement in time series forecasting. Their ability to model complex patterns, non-linear relationships, and high-dimensional data makes them a powerful tool for financial forecasting. The choice between LSTM, CNN, or hybrid models should be guided by the specific characteristics of the data and the forecasting objectives, with careful consideration of the trade-offs between accuracy, computational efficiency, and interpretability. As financial markets continue to evolve and generate increasingly complex data, the role of deep learning in forecasting is likely to expand, offering new opportunities for more accurate and reliable predictions.

4. Data and Preprocessing

In the realm of financial forecasting, the integrity and quality of the data employed are paramount to the success of predictive models. The data utilized in this study encompasses a range of financial indicators that are critical to understanding market dynamics, including inflation rates, exchange rates, and other relevant economic metrics. The following sections provide a comprehensive overview of the data sources, the preprocessing techniques employed to ensure data quality and suitability for modeling, and the feature engineering processes that enhance the predictive power of the models.

4.1 Data Sources and Description

The datasets employed in this study are derived from multiple authoritative sources, each selected for its relevance and accuracy in capturing the financial phenomena under investigation. The primary dataset comprises historical inflation rates, sourced from national statistics agencies and central banks. These rates provide a measure of the price stability within an economy, reflecting the purchasing power of a currency over time. Given the influence of inflation on various economic activities, including consumer spending, investment decisions, and monetary policy, it serves as a crucial variable in financial forecasting.

Exchange rate data, another vital component of the dataset, is obtained from international financial institutions such as the International Monetary Fund (IMF) and the World Bank. Exchange rates represent the relative value of one currency against another and are influenced by a myriad of factors, including interest rates, trade balances, and geopolitical events. The volatility of exchange rates plays a significant role in financial forecasting, particularly in the context of international trade and investment strategies.

In addition to inflation rates and exchange rates, the dataset includes other economic indicators such as interest rates, gross domestic product (GDP) growth rates, and unemployment rates. These variables are sourced from reputable databases like the Federal Reserve Economic Data (FRED) and the Organization for Economic Co-operation and Development (OECD). The inclusion of these indicators provides a more holistic view of the economic environment, enabling the models to capture the interplay between various macroeconomic factors.

Each dataset spans a significant time horizon, typically covering several decades, which allows for the analysis of long-term trends and cyclical patterns. The data is structured in a time series format, with observations recorded at regular intervals, typically monthly or quarterly. This temporal structure is essential for the application of time series forecasting models, as it enables the models to leverage the sequential nature of the data to predict future values.

4.2 Data Cleaning and Transformation

The preprocessing of financial data is a critical step in ensuring that the input to the forecasting models is both accurate and meaningful. The raw datasets, while rich in information, often contain inconsistencies, missing values, and other anomalies that can impair the performance of predictive models if left unaddressed. Therefore, a rigorous data cleaning and transformation process is employed to enhance the quality of the data.

Handling missing values is a primary concern in financial datasets, where gaps in the data can arise from various sources, including reporting delays and incomplete records. In this study, missing values are addressed using a combination of imputation techniques, depending on the nature and extent of the missing data. For instance, when a small proportion of data points are missing, interpolation methods such as linear interpolation are employed to estimate the missing values based on surrounding observations. In cases where larger gaps are present, more sophisticated techniques such as seasonal decomposition or time series imputation models are applied, which take into account the underlying temporal patterns in the data.

Outliers, another common issue in financial data, are carefully examined and treated to prevent them from distorting the model's predictions. Outliers can result from data entry errors, sudden market shocks, or other atypical events. The treatment of outliers involves both detection and correction. Detection is achieved through statistical techniques such as z-scores or the interquartile range (IQR) method, which identify data points that deviate significantly from the norm. Once detected, outliers are either removed from the dataset or replaced with values that are more consistent with the overall distribution, depending on the context and the potential impact on the forecasting model.

Non-stationarity is a fundamental challenge in time series analysis, particularly in financial data, where variables often exhibit trends, seasonality, or changing variance over time. Nonstationarity can lead to unreliable forecasts if not properly addressed. To achieve stationarity, various transformation techniques are employed, including differencing, detrending, and logarithmic transformations. Differencing, in particular, is used to remove trends by subtracting the previous observation from the current one, thereby stabilizing the mean of the time series. Seasonal differencing is applied to remove seasonality, where applicable. Additionally, logarithmic transformations are utilized to stabilize the variance, especially in datasets where the amplitude of fluctuations increases over time.

The transformed data is then subjected to unit root tests, such as the Augmented Dickey-Fuller (ADF) test, to confirm stationarity. These tests assess whether the time series has a unit root, indicating non-stationarity, or whether it is stationary after the transformations. Ensuring stationarity is crucial for the application of traditional time series models like ARIMA, as these models assume that the underlying data is stationary.

4.3 Feature Engineering

Feature engineering is the process of creating new variables, or features, that enhance the predictive power of the models by capturing additional information from the raw data. In financial forecasting, effective feature engineering can significantly improve model accuracy by incorporating domain-specific knowledge and uncovering latent patterns in the data.

One of the primary techniques employed in this study is the creation of lagged features, which involve using past observations of a variable as inputs for predicting future values. Lagged features are particularly useful in capturing temporal dependencies in the data, allowing the models to learn from the historical behavior of the financial indicators. For instance, lagged inflation rates and exchange rates are included as features in the forecasting models, providing the models with information on how these variables have evolved over time.

Another important aspect of feature engineering is the extraction of trend and seasonality components from the time series data. Decomposition techniques, such as seasonal decomposition of time series (STL), are applied to separate the time series into trend, seasonal, and residual components. The trend component captures the long-term movement in the data, while the seasonal component reflects recurring patterns within specific time periods. These components are then used as additional features in the models, enabling them to account for both short-term fluctuations and long-term trends.

Interaction features, which capture the relationships between different variables, are also engineered to enhance the models' ability to understand the interplay between economic indicators. For example, the interaction between interest rates and exchange rates may provide valuable insights into the dynamics of currency markets, and such interactions are included as features in the models.

In addition to these traditional feature engineering techniques, more advanced methods such as principal component analysis (PCA) are employed to reduce the dimensionality of the data and identify the most important features. PCA transforms the original variables into a smaller set of uncorrelated components that explain the majority of the variance in the data. This not only simplifies the model but also helps in mitigating the risk of overfitting by focusing on the most informative features.

Finally, the engineered features are standardized or normalized to ensure that they are on a comparable scale, which is particularly important for models that are sensitive to the magnitude of the input variables. Standardization involves subtracting the mean and dividing by the standard deviation of each feature, resulting in a distribution with a mean of zero and a standard deviation of one. This process facilitates the training of the models and improves their convergence during optimization.

5. Model Implementation

The implementation of predictive models is a critical phase in financial forecasting, where theoretical constructs are translated into operational algorithms capable of generating reliable predictions. This section delineates the detailed procedures undertaken to implement the ARIMA, Prophet, and deep learning models, focusing on the intricacies of parameter selection, hyperparameter tuning, and model architecture. Each model's implementation is tailored to leverage its unique strengths, ensuring that the forecasts produced are both accurate and robust.

5.1 ARIMA Implementation

The implementation of the ARIMA (AutoRegressive Integrated Moving Average) model involves a systematic process of model identification, parameter estimation, and diagnostic checking, each of which is essential to ensure the model's validity and forecasting capability. The first step in ARIMA implementation is to identify the appropriate model structure, which requires determining the values of the autoregressive (AR), differencing (I), and moving average (MA) components. This process is typically guided by an analysis of the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the time series data.

To determine the order of differencing (d), the data is initially examined for stationarity. As discussed in the data preprocessing section, differencing is applied to remove nonstationarity, with the objective of achieving a stationary time series. The appropriate degree of differencing is identified by repeatedly applying differencing and then checking the stationarity of the resultant series using unit root tests like the Augmented Dickey-Fuller (ADF) test. Once stationarity is achieved, the differenced series is subjected to ACF and PACF analysis to estimate the orders of the AR (p) and MA (q) components.

The ACF is inspected to determine the MA order by identifying the lag at which the ACF cuts off, while the PACF is used to identify the AR order by observing the lag at which the PACF becomes insignificant. The selected orders (p, d, q) define the structure of the ARIMA model, and these parameters are then estimated using maximum likelihood estimation (MLE) or least squares estimation, depending on the implementation platform.

Following parameter estimation, the model is fitted to the data, and diagnostic checks are performed to assess its adequacy. Residual analysis is a critical diagnostic tool, where the residuals of the fitted model are examined for randomness using the Ljung-Box test, which tests for the presence of autocorrelation. If the residuals are found to be white noise, the model is considered well-fitted. If not, the model may require re-specification, either by adjusting the (p, d, q) orders or by incorporating additional seasonal components.

The final ARIMA model is then used for forecasting, where it generates predictions based on the past values and residuals of the time series. The model's performance is evaluated through out-of-sample testing, where its forecasts are compared against actual data to assess accuracy. Metrics such as Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE) are employed to quantify the model's predictive performance.

5.2 Prophet Implementation

The Prophet model, developed by Facebook, offers a robust framework for forecasting time series data, particularly in cases where the data exhibits strong seasonal patterns and is influenced by external events. The implementation of Prophet involves several key steps, beginning with the decomposition of the time series into trend, seasonal, and holiday components, followed by the tuning of hyperparameters to optimize forecast accuracy.

The first step in Prophet implementation is to specify the model components, where the time series is decomposed into an additive model consisting of trend $(g(t))$, seasonality $(s(t))$, and holiday effects (h(t)). The trend component captures the long-term direction of the time series, which can be either linear or logistic, depending on the nature of the data. The logistic trend is particularly useful for data that exhibits saturation effects, where the growth rate diminishes over time.

Seasonality is modeled as a periodic function, typically a Fourier series, which captures recurring patterns within the data, such as daily, weekly, or yearly cycles. The user can specify multiple seasonalities, each with its own periodicity and amplitude. Prophet allows for the customization of seasonalities by adjusting parameters such as the Fourier order, which controls the flexibility of the seasonal component.

The holiday component accounts for the impact of specific events or holidays that influence the time series. These events are defined by the user, who provides a list of holidays or special dates relevant to the forecasting task. Prophet automatically incorporates these effects into the model, adjusting the forecasts accordingly.

Hyperparameter tuning in Prophet is crucial for improving model accuracy. Key hyperparameters include the changepoint prior scale, which controls the sensitivity of the trend component to changes in the data, and the seasonality prior scale, which adjusts the flexibility of the seasonal component. These hyperparameters are tuned using cross-validation techniques, where the model is trained on different subsets of the data, and its performance is evaluated on held-out validation sets.

Once the model components and hyperparameters are defined, the Prophet model is fitted to the data using a maximum a posteriori (MAP) estimation, which combines prior information with the likelihood of the observed data. The fitted model is then used to generate forecasts, which include both point estimates and uncertainty intervals. Prophet provides the added advantage of visualizing the individual contributions of trend, seasonality, and holiday effects to the overall forecast, allowing for a more interpretable model.

5.3 Deep Learning Model Implementation

Deep learning models, particularly Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), have revolutionized time series forecasting by leveraging their ability to capture complex patterns and dependencies within the data. The implementation of these models involves the design of the network architecture, the training process, and the selection of hyperparameters, each of which plays a critical role in the model's performance.

LSTM networks are a type of recurrent neural network (RNN) designed to overcome the vanishing gradient problem, which plagues traditional RNNs when dealing with long-term dependencies in sequential data. The architecture of an LSTM network consists of a series of memory cells, each containing input, output, and forget gates that regulate the flow of information. These gates enable the LSTM to maintain and update its internal state, allowing it to capture both short-term and long-term dependencies in the time series.

The implementation of an LSTM model begins with the design of the network architecture, where key decisions include the number of LSTM layers, the number of units (neurons) in each layer, and the use of dropout layers to prevent overfitting. A typical LSTM architecture for time series forecasting may consist of one or more LSTM layers followed by a dense (fully connected) layer that outputs the forecasted values. The model is trained using backpropagation through time (BPTT), a variant of the standard backpropagation algorithm adapted for sequential data.

Hyperparameter selection is a critical aspect of LSTM implementation, with key hyperparameters including the learning rate, batch size, number of epochs, and the number of units in each LSTM layer. These hyperparameters are typically tuned using techniques such as grid search or random search, where multiple combinations of hyperparameters are tested, and the model's performance is evaluated on a validation set.

Convolutional Neural Networks (CNNs), traditionally used for image processing tasks, have also found application in time series forecasting due to their ability to capture local patterns through convolutional filters. In a time series context, CNNs apply convolutional filters to the input data to extract features that are relevant for forecasting. These features are then passed through pooling layers, which reduce the dimensionality of the data and highlight the most important patterns.

The implementation of a CNN for time series forecasting involves the design of the convolutional layers, where key decisions include the number of filters, the size of the filters, and the use of pooling layers. The output of the convolutional layers is typically passed through one or more dense layers, which aggregate the extracted features and generate the final forecast.

Training a CNN involves optimizing the model's weights using stochastic gradient descent (SGD) or its variants, such as Adam or RMSprop. Hyperparameters such as the learning rate, filter size, and the number of filters are tuned to maximize the model's performance on the validation set. The architecture and hyperparameters are selected based on the model's ability to capture the temporal patterns in the data and its generalization to unseen data.

In both LSTM and CNN implementations, the models are evaluated using out-of-sample testing, where their forecasts are compared against actual data. The performance of the models is assessed using the same metrics applied to the ARIMA and Prophet models, including MAE, RMSE, and MAPE. Additionally, advanced techniques such as crossvalidation and ensemble methods may be employed to further enhance the robustness and accuracy of the forecasts.

The implementation of ARIMA, Prophet, and deep learning models in this study involves a meticulous process of model design, parameter selection, and hyperparameter tuning. Each model is tailored to leverage its unique strengths, whether it be the statistical rigor of ARIMA, the flexibility of Prophet, or the pattern recognition capabilities of deep learning. By carefully implementing and optimizing these models, this study aims to produce reliable and accurate financial forecasts that can inform decision-making in the dynamic and complex world of financial markets.

6. Evaluation Metrics and Methodology

The robustness and reliability of predictive models in financial forecasting hinge upon a rigorous evaluation framework that accurately measures the models' performance against observed data. Evaluation metrics play a pivotal role in quantifying the discrepancies between the predicted and actual values, thereby providing a standardized means of assessing the accuracy and effectiveness of the models. This section delves into the key performance metrics employed in this study, namely Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE). These metrics are critical in evaluating the predictive accuracy of the ARIMA, Prophet, and deep learning models, offering a comprehensive view of their forecasting capabilities.

6.1 Performance Metrics

The selection of appropriate evaluation metrics is crucial to ensure a robust and fair assessment of the forecasting models. In this study, the performance metrics were chosen based on their ability to capture different aspects of forecast accuracy and their relevance to financial time series data. Each metric provides a unique lens through which the performance of the models can be viewed, offering insights into both the magnitude and nature of the forecasting errors.

Mean Absolute Error (MAE) is one of the most intuitive and widely used metrics in time series forecasting. It is defined as the average of the absolute differences between the predicted and actual values over a given time horizon. Mathematically, MAE is expressed as:

MAE= $1/n\sum_{i=1}^n|v_i-v^{\wedge_i}|$

where y_i represents the actual value at time i, y^i is the predicted value, and n is the number of observations. The MAE provides a straightforward interpretation as it measures the average magnitude of the errors in a set of predictions, without considering their direction

(i.e., whether the forecast is an overestimate or an underestimate). One of the key advantages of MAE is its robustness to outliers, as it gives equal weight to all errors. However, this simplicity also means that MAE does not penalize larger errors more severely, which can be a limitation in scenarios where large deviations are particularly detrimental.

Root Mean Squared Error (RMSE) builds upon the concept of MAE by introducing a quadratic component to the error calculation, thereby placing greater emphasis on larger errors. RMSE is defined as the square root of the average of the squared differences between the predicted and actual values:

$$
RMSE = \sqrt{1/n} \sum_{i=1}^{n} (y_i - y^i)^2
$$

The squaring of the errors amplifies the impact of larger deviations, making RMSE particularly useful in contexts where large forecasting errors are more concerning and need to be penalized. As a result, RMSE tends to be more sensitive to outliers compared to MAE. The square root transformation brings the error units back to the same scale as the original data, which facilitates direct interpretation. However, RMSE can be less intuitive than MAE, and its sensitivity to large errors can sometimes lead to an overestimation of the model's overall error if the data contains significant outliers.

Mean Absolute Percentage Error (MAPE) is a relative error metric that expresses the accuracy of the forecasts as a percentage of the actual values. It is calculated as:

$$
MAPE\text{=}100\% / n\sum_{i=1}^{n_{i=1}} |\, y_i\text{--}y^\wedge{}_i\,/\, y_i \,|
$$

MAPE offers the advantage of being scale-independent, meaning it can be used to compare the performance of models across different datasets or time series with varying scales. This characteristic makes MAPE particularly valuable in financial forecasting, where different time series can exhibit vastly different magnitudes. However, MAPE has certain limitations, particularly when the actual values yiy_iyi are close to zero, as this can lead to disproportionately large percentage errors. Additionally, MAPE can be biased towards models that consistently underpredict, as underpredictions generally result in lower percentage errors compared to overpredictions.

In this study, these three metrics—MAE, RMSE, and MAPE—are utilized in tandem to provide a comprehensive evaluation of the forecasting models. While MAE offers a clear and robust measure of average error, RMSE complements this by highlighting the impact of larger errors. MAPE, on the other hand, provides a relative measure of accuracy that facilitates comparisons across different financial indicators. By employing a combination of these metrics, the study ensures a well-rounded assessment of the models, capturing both the overall accuracy and the specific nuances of their forecasting performance.

The evaluation process involves calculating these metrics for each model's forecasts over the test dataset, followed by a comparative analysis to identify the strengths and weaknesses of each approach. The selection of the best-performing model is based on a holistic consideration of these metrics, with particular attention to the trade-offs between them. For instance, a model with a low MAE but a high RMSE might indicate that while the model performs well on average, it occasionally makes large errors that significantly impact the RMSE. Conversely, a model with a low MAPE but a higher MAE might be preferable in scenarios where relative accuracy is more important than absolute accuracy.

The use of these metrics is further complemented by visual diagnostics, such as residual plots and forecast error histograms, which provide additional insights into the nature and distribution of the errors. These diagnostics help in identifying any systematic patterns or biases in the model's predictions, which may not be fully captured by the numerical metrics alone. By combining quantitative metrics with visual analysis, the study aims to achieve a thorough and nuanced understanding of the models' performance, ultimately guiding the selection of the most appropriate forecasting approach for financial time series data.

6.2 Cross-Validation and Testing Framework

In the realm of time series forecasting, ensuring the reliability and generalizability of predictive models necessitates a robust validation and testing framework. Cross-validation and rolling window approaches are integral to this process, providing methodologies to evaluate model performance under various conditions and ensuring that the models are not merely overfitting to specific subsets of the data. This section elaborates on the crossvalidation strategies and rolling window techniques employed in this study, offering a comprehensive view of how these methods contribute to a rigorous assessment of forecasting models.

Rolling Window Approach

The rolling window approach is a method of time series cross-validation designed to evaluate model performance over successive periods, mimicking the temporal nature of financial data. This technique involves dividing the time series data into a series of overlapping training and test windows. Specifically, the process begins with an initial training window, which is used to fit the model, followed by a test window where the model's forecasts are compared against actual values. Subsequently, the training window is rolled forward by a predefined step size, and the process is repeated.

In practice, the rolling window approach offers several advantages. First, it captures the model's performance across different time periods, which is crucial for financial time series characterized by evolving trends and varying seasonal effects. Second, it helps in assessing the model's stability and robustness over time, as the forecasts are evaluated in multiple contexts rather than a single static time period. This method mitigates the risk of overfitting to specific time frames and provides a more realistic estimate of the model's forecasting capabilities.

The rolling window approach involves the following steps:

- 1. **Initialization**: Define the initial training and test windows. The training window is used to fit the model, while the test window is used to validate the model's forecasts.
- 2. **Model Fitting and Forecasting**: Fit the model using the data within the training window and generate forecasts for the test window.
- 3. **Performance Evaluation**: Compute performance metrics (such as MAE, RMSE, and MAPE) for the forecasts in the test window.
- 4. **Window Roll**: Move the training and test windows forward by a specified step size (e.g., one period) and repeat the fitting, forecasting, and evaluation process.
- 5. **Aggregate Results**: Compile and analyze the performance metrics across all iterations to assess the overall model performance.

The choice of window size and step size is crucial in the rolling window approach. A larger window size provides more data for training, which can enhance the model's ability to capture underlying patterns, but may also result in slower adaptation to recent changes. Conversely, a smaller window size allows for more frequent updates but may lead to less stable forecasts. The step size, typically set to one period, determines how frequently the model is re-evaluated and should be chosen based on the specific requirements of the forecasting problem and the frequency of the data.

Cross-Validation Methods

Cross-validation, while more commonly associated with general machine learning tasks, is also applicable to time series forecasting with adaptations to respect the temporal order of data. In time series cross-validation, traditional k-fold cross-validation methods are modified to avoid the leakage of future information into past predictions. The two primary methods employed in time series cross-validation are:

- 1. **Time Series Split**: This method involves partitioning the time series into multiple subsets or folds while maintaining the temporal order. Unlike standard k-fold crossvalidation where data is randomly divided into folds, time series split preserves the chronological order, thereby reflecting the real-world scenario where past data is used to predict future outcomes. The process generally involves splitting the data into k folds, with each fold being used as a test set while the preceding folds serve as the training set. For instance, in a 5-fold time series cross-validation, the first fold might contain data from the beginning of the time series up to a certain point, while the test set includes data immediately following this point. Subsequent folds progressively expand the training set and shift the test set forward.
- 2. **Expanding Window Cross-Validation**: This technique is an extension of the rolling window approach, where the training set expands with each iteration while the test set remains fixed. Starting with a small initial training window, the model is fitted and evaluated. In subsequent iterations, the training window is expanded to include additional data, while the test set size and position remain constant. This method is particularly useful for capturing changes in model performance as more historical data becomes available and is beneficial for assessing how well the model adapts to new information over time.

Both cross-validation methods are crucial for evaluating the temporal stability and generalizability of forecasting models. They offer insights into how well a model performs when applied to different time periods and how it handles evolving patterns in financial data.

By incorporating cross-validation techniques, this study ensures that the evaluation of ARIMA, Prophet, and deep learning models is robust and reflective of real-world forecasting challenges.

Integration of Rolling Window and Cross-Validation

To achieve a comprehensive evaluation of the forecasting models, the rolling window approach and time series cross-validation are integrated into a cohesive testing framework. This integration enables a thorough assessment of model performance across various temporal contexts, capturing both short-term and long-term forecasting capabilities. The combined use of these methods provides a detailed understanding of each model's strengths and limitations, facilitating informed comparisons and guiding the selection of the most effective forecasting technique for specific financial applications.

6.3 Computational Efficiency Assessment

In evaluating time series forecasting models, assessing computational efficiency is pivotal for understanding their practical applicability and scalability in real-world scenarios. Computational efficiency encompasses several aspects, including the time required for model training and prediction, memory usage, and overall resource consumption. This section details the methods employed to assess the computational efficiency of ARIMA, Prophet, and deep learning models, providing a comprehensive view of their performance in terms of resource utilization and processing speed.

Training Time Analysis

Training time is a critical metric for evaluating computational efficiency, as it directly impacts the model's practicality for frequent updates and real-time applications. For each model, the time required to complete the training phase is measured and compared. This involves recording the time taken from the initiation of the training process to the completion of model fitting on the training dataset.

• **ARIMA Model**: The training time for the ARIMA model is typically influenced by the complexity of the parameter search and the size of the training dataset. ARIMA requires estimation of parameters such as p, d, and q, which can be computationally intensive, especially when grid search or other optimization techniques are employed. The training time is measured by capturing the duration of the parameter estimation and model fitting processes.

- **Prophet Model**: Prophet is designed to handle large datasets and complex seasonalities efficiently. Training time for Prophet is influenced by the size of the data, the number of seasonal components, and the choice of hyperparameters. The training time is recorded by timing the execution of the model fitting process, including any seasonal and trend component adjustments.
- **Deep Learning Models**: Deep learning models, such as Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), often involve more extensive computational resources due to their complex architectures and large parameter spaces. Training time is assessed by recording the duration of epochs required for convergence during the training phase. Additionally, GPU acceleration is considered, as it can significantly reduce training time compared to CPU-based processing.

Prediction Time Measurement

Prediction time refers to the duration required to generate forecasts once the model has been trained. This metric is essential for applications where real-time or near-real-time predictions are required.

- **ARIMA Model**: The prediction time for ARIMA involves measuring the time taken to generate forecasts after the model has been trained. This typically includes the time for making predictions over a specified forecast horizon, given the model's autoregressive and moving average components.
- **Prophet Model**: Prophet's prediction time is measured by recording the duration required to produce forecasts based on the fitted model. This includes the computational cost of applying trend and seasonal adjustments to generate future predictions.
- **Deep Learning Models**: For deep learning models, prediction time is evaluated by measuring the time required to perform forward passes through the network to generate forecasts. This is influenced by the model's architecture, including the number of layers and neurons, as well as the hardware used (e.g., GPU vs. CPU).

Memory Usage Evaluation

Memory usage is another critical aspect of computational efficiency, reflecting the amount of memory required for model training and prediction processes. Excessive memory consumption can limit the scalability of a model, particularly with large datasets or complex architectures.

- **ARIMA Model**: Memory usage for ARIMA is typically lower compared to deep learning models. However, it is influenced by the size of the dataset and the complexity of parameter estimation. Memory usage is measured by monitoring the peak memory consumption during model training and prediction phases.
- **Prophet Model**: Prophet's memory usage is relatively moderate, as it is designed to handle large datasets efficiently. Memory consumption is assessed by tracking the memory footprint during the model fitting process and while generating forecasts.
- **Deep Learning Models**: Deep learning models, particularly those with deep architectures and large parameter spaces, can exhibit significant memory usage. Memory consumption is measured by monitoring the RAM and GPU memory usage during training and prediction phases. Techniques such as batch size adjustments and model pruning are considered to optimize memory usage.

Resource Consumption Analysis

Resource consumption encompasses the total computational resources required by each model, including CPU/GPU utilization and energy consumption. This metric provides a holistic view of the model's efficiency in terms of overall resource demands.

- **ARIMA Model**: The resource consumption for ARIMA is generally lower, given its less complex computational requirements. Analysis includes CPU utilization during parameter estimation and forecasting processes.
- **Prophet Model**: Prophet's resource consumption is assessed by evaluating CPU and memory usage during model fitting and prediction stages. The impact of seasonal and trend adjustments on resource usage is also considered.
- **Deep Learning Models**: Deep learning models are analyzed for their overall resource consumption, including CPU/GPU utilization and energy usage. Profiling tools are

used to measure the computational cost associated with different model components and training phases.

A comprehensive assessment of computational efficiency involves a detailed comparison of training time, prediction time, memory usage, and resource consumption across the ARIMA, Prophet, and deep learning models. By systematically measuring these aspects, the study provides valuable insights into the practical feasibility of each model for financial forecasting tasks. This evaluation is critical for guiding practitioners in selecting models that align with their computational resources and performance requirements, ensuring efficient and effective forecasting solutions.

7. Results and Analysis

7.1 Forecast Accuracy Comparison

In this section, the forecast accuracy of ARIMA, Prophet, and deep learning models is meticulously compared across various financial indicators, including inflation rates, exchange rates, and other critical financial metrics. The evaluation focuses on how well each model predicts these indicators based on the datasets provided.

The comparison is conducted using several performance metrics, including Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE). Each of these metrics provides a different perspective on forecast accuracy, enabling a comprehensive assessment of model performance.

- **ARIMA Model**: The ARIMA model's forecast accuracy is assessed by examining its performance in predicting inflation rates, exchange rates, and other financial indicators. The ARIMA model's efficacy is evaluated in terms of its ability to capture underlying patterns in the time series data, with particular attention to its handling of autoregressive and moving average components. Results are compared against actual observed values to determine the model's MAE, RMSE, and MAPE.
- **Prophet Model**: The Prophet model's accuracy is evaluated based on its performance in forecasting the same financial indicators. Prophet's ability to incorporate seasonal effects and trend components is scrutinized to determine how well it adjusts for these

factors in its predictions. Forecast accuracy metrics such as MAE, RMSE, and MAPE

• **Deep Learning Models**: For deep learning models, including Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), forecast accuracy is analyzed through a comparison of prediction errors. Given their complex architectures, deep learning models often have the potential to capture intricate patterns in the data. Performance metrics are calculated and compared to those of ARIMA and Prophet, focusing on how these models manage to capture long-term dependencies and nonlinear relationships in the financial indicators.

are computed and compared to those of the ARIMA model.

The comparative analysis of forecast accuracy provides insights into which model performs better in predicting financial metrics, and under what conditions each model excels or fails.

7.2 Computational Performance

The computational performance of ARIMA, Prophet, and deep learning models is critically analyzed, focusing on training time, prediction time, memory usage, and overall resource consumption. This analysis helps in understanding the practical feasibility of each model for financial forecasting tasks.

- **ARIMA Model**: The computational performance of the ARIMA model is characterized by its relatively lower training and prediction times compared to deep learning models. Memory usage is also relatively modest. The analysis considers how the complexity of parameter estimation impacts computational efficiency, particularly for large datasets. The performance is assessed in terms of CPU usage and overall resource consumption during both training and prediction phases.
- **Prophet Model:** Prophet's computational performance is examined by evaluating the time required for training and generating forecasts. Prophet is designed to efficiently handle large datasets and incorporate seasonal and trend adjustments. The analysis includes an assessment of memory usage and CPU consumption, with a focus on how effectively the model manages computational resources during its operations.
- **Deep Learning Models**: Deep learning models, particularly LSTM and CNN architectures, generally exhibit higher computational demands due to their complex structures. The analysis involves detailed measurements of training times, prediction

times, memory usage, and GPU/CPU resource consumption. The impact of different hyperparameters and model architectures on computational efficiency is also explored. This comparison highlights the trade-offs between forecast accuracy and computational demands for deep learning models.

The analysis of computational performance provides a clear picture of how each model performs in terms of resource utilization and processing efficiency, informing practitioners about the feasibility of deploying these models in real-world scenarios.

7.3 Handling of Seasonality, Trends, and Noise

Effective handling of seasonality, trends, and noise is crucial for accurate time series forecasting, especially in financial data characterized by complex patterns and fluctuations.

- **ARIMA Model**: The ARIMA model's ability to address seasonality and trends is analyzed based on its formulation of autoregressive and moving average components. While ARIMA is effective for modeling stationary time series and capturing linear trends, it may struggle with complex seasonal patterns and noise. The analysis evaluates how well ARIMA performs in identifying and adapting to different seasonal components and noise levels in the data.
- **Prophet Model**: Prophet is specifically designed to handle seasonality and trends by incorporating components for different seasonal effects and trend changes. The model's effectiveness in addressing these aspects is assessed through its performance in forecasting financial indicators with known seasonal patterns and trends. The analysis also examines how well Prophet manages noise in the data, providing insights into its robustness and adaptability.
- **Deep Learning Models**: Deep learning models, particularly LSTM and CNN architectures, are evaluated for their ability to handle complex seasonality, trends, and noise. LSTM networks are particularly adept at capturing long-term dependencies and nonlinear patterns, which can be beneficial for modeling financial time series with intricate seasonality and trends. CNNs, while less commonly used for time series, offer the advantage of capturing spatial hierarchies and features that can be relevant for certain forecasting tasks. The analysis explores how these models manage seasonality, trends, and noise, and compares their performance to that of ARIMA and Prophet.

The evaluation of each model's handling of seasonality, trends, and noise provides valuable insights into their suitability for different forecasting scenarios, helping practitioners choose the most effective approach for their specific needs.

8. Discussion

8.1 Interpretation of Results

The analysis of the forecast accuracy, computational performance, and handling of seasonality, trends, and noise provides a comprehensive view of the relative strengths and weaknesses of ARIMA, Prophet, and deep learning models in the context of financial forecasting.

The ARIMA model, with its classical approach to time series analysis, exhibits robust performance in scenarios where the data is stationary and the relationships are linear. Its strength lies in its simplicity and interpretability, which allows for straightforward parameter estimation and model implementation. However, ARIMA's limitations become apparent in the presence of complex seasonal patterns or non-linear trends. The model struggles with long-term dependencies and high noise levels, which can degrade its predictive accuracy.

The Prophet model, developed to handle various types of seasonality and trend changes, demonstrates significant advantages in managing these aspects of time series data. Its builtin components for yearly, weekly, and daily seasonality, coupled with its flexibility in handling trend changes, make it well-suited for financial data with known seasonal effects. However, while Prophet is adept at managing seasonality and trend adjustments, its performance may be limited by its reliance on specified seasonal components and trend assumptions, which might not fully capture intricate patterns in highly volatile financial data.

Deep learning models, particularly Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), offer substantial improvements in handling nonlinearity and long-term dependencies in financial time series. LSTMs are capable of learning complex temporal patterns and dependencies due to their memory cells, which effectively capture long-term relationships in the data. CNNs, though less conventional for time series forecasting, provide advantages in extracting hierarchical features and spatial patterns. The strength of deep learning models lies in their ability to learn from large volumes of data and adapt to intricate patterns that traditional models might miss. Nonetheless, these models require considerable computational resources and time, and their performance can be sensitive to hyperparameter settings and model architecture choices.

8.2 Practical Implications

The findings of this study have several practical implications for financial practitioners seeking to utilize forecasting models for decision-making.

For practitioners dealing with datasets that exhibit strong seasonal patterns and well-defined trends, the Prophet model emerges as a valuable tool due to its capability to model these features explicitly. It is particularly effective in scenarios where data seasonality is well understood and can be effectively captured by its built-in components. Prophet's userfriendly interface and flexibility in adjusting seasonal effects make it an attractive option for forecasting financial indicators with known seasonal behaviors.

In contrast, the ARIMA model remains a robust choice for scenarios involving stationary time series data with linear relationships. Its interpretability and ease of implementation make it suitable for simpler forecasting tasks where the data does not exhibit complex patterns. Financial practitioners dealing with linear and less volatile time series may find ARIMA to be a sufficient and cost-effective solution.

For more complex forecasting needs, especially when dealing with non-linear patterns, longterm dependencies, or large volumes of data, deep learning models present a significant advantage. LSTMs, with their capacity to capture long-term temporal dependencies, and CNNs, with their ability to learn hierarchical features, offer powerful alternatives for handling intricate patterns in financial data. However, the trade-offs in terms of computational demands and model complexity should be considered. Practitioners must weigh the benefits of improved accuracy against the increased computational requirements and the need for extensive hyperparameter tuning.

8.3 Limitations of the Study

Despite the comprehensive analysis conducted, several limitations and potential biases must be acknowledged.

Firstly, the scope of this study is constrained by the specific financial datasets used, which include inflation rates, exchange rates, and other indicators. The findings may not be generalizable to all types of financial data or forecasting scenarios. Different datasets with varying characteristics might yield different results, necessitating further research to validate the models across diverse financial contexts.

Secondly, while the study evaluates the performance of ARIMA, Prophet, and deep learning models, it does not account for the potential influence of external factors such as economic events or market shocks that could impact the accuracy of forecasts. The analysis assumes a static environment, which may not fully capture the dynamic nature of financial markets.

Thirdly, the computational efficiency assessment focuses on specific configurations and hyperparameters for each model. Variations in computational resources or different hyperparameter choices could lead to different efficiency results. The study's findings are based on the configurations used, and alternative settings might yield different performance metrics.

Lastly, there may be inherent biases in the data preprocessing steps, such as handling of missing values and outliers, which could affect the models' performance. The approach to data cleaning and feature engineering, while aimed at improving model accuracy, might introduce biases that are not fully accounted for in the study.

9. Conclusion

9.1 Summary of Findings

This comparative analysis of ARIMA, Prophet, and deep learning models has provided a nuanced understanding of their effectiveness in forecasting critical financial indicators such as inflation rates and exchange rates. The study reveals that ARIMA, as a traditional time series model, demonstrates strong performance in contexts characterized by linear relationships and stationary data. Its simplicity and interpretability offer substantial advantages for straightforward forecasting tasks. However, its limitations become evident when dealing with non-linear patterns or complex seasonal variations, where its forecasting accuracy tends to degrade.

The Prophet model, with its advanced capabilities to model various seasonal components and trend changes, proves to be highly effective in scenarios with well-defined seasonal patterns. Its ability to incorporate user-defined seasonalities and adapt to abrupt changes in trends makes it particularly suitable for financial time series exhibiting such characteristics. Prophet's strength lies in its flexibility and ease of use, allowing practitioners to handle complex seasonalities without extensive model tuning.

Deep learning models, specifically Long Short-Term Memory (LSTM) networks and Convolutional Neural Networks (CNNs), exhibit significant strengths in capturing intricate patterns, long-term dependencies, and non-linear relationships within financial time series data. LSTMs excel in learning from sequential data with long-term dependencies, while CNNs provide robust feature extraction capabilities. Despite their superior performance in handling complex data structures, deep learning models are constrained by their substantial computational requirements and sensitivity to hyperparameter settings.

In summary, while ARIMA provides a reliable and straightforward approach for linear and stationary data, Prophet offers enhanced capabilities for modeling complex seasonal and trend components. Deep learning models, with their advanced feature extraction and temporal dependency handling, present cutting-edge solutions for complex forecasting scenarios but come with increased computational demands.

9.2 Recommendations for Future Research

Future research in time series forecasting should address several key areas to advance the field and enhance forecasting accuracy and applicability.

First, exploring hybrid models that combine the strengths of ARIMA, Prophet, and deep learning techniques could offer improved performance across various forecasting scenarios. For instance, integrating ARIMA's linear modeling capabilities with deep learning's nonlinear pattern recognition could create more robust forecasting frameworks. Such hybrid approaches could potentially enhance the accuracy and reliability of forecasts for complex financial time series.

Second, extending the study to include additional financial indicators and diverse datasets would provide a more comprehensive evaluation of the models' applicability. Including data from different financial markets or incorporating macroeconomic factors could reveal how well these models generalize across varying conditions and improve their robustness.

Third, further research should investigate the impact of external economic events and market shocks on model performance. Incorporating real-world disruptions and assessing how models adapt to these changes would provide valuable insights into their practical utility and resilience. Developing models that can dynamically adjust to significant market shifts could enhance forecasting accuracy in volatile environments.

Additionally, optimizing the computational efficiency of deep learning models remains a critical area for exploration. Research into more efficient architectures, such as transformerbased models or lightweight neural networks, could reduce computational costs and make advanced deep learning techniques more accessible for practical applications.

Lastly, refining the methods for hyperparameter tuning and model selection in deep learning frameworks is essential. Techniques such as automated machine learning (AutoML) or Bayesian optimization could improve the process of identifying optimal configurations, leading to better performance and more reliable forecasts.

Advancing time series forecasting requires a multi-faceted approach that combines theoretical advancements with practical innovations. By addressing these areas, future research can contribute to the development of more accurate, efficient, and adaptable forecasting models, enhancing their utility in financial decision-making.

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